Time-Resolved Digital Interferometry for High Speed Flow Visualization in Hypersonic Shock Tunnel

G.M. Hegde2*, G. Jagadeesh1 and K.P.J. Reddy1

Abstract | In this article we report on the development of time-resolved digital interferometric visualization technique integrated with short duration (1 ms) hypersonic shock tunnel. Dynamics of the Mach 6 flow field around 40° blunt cone model is visualized using a combination of cw laser, Mach-Zehnder interferometer and a very high-speed digital camera. Digital interferograms with time resolution of 139 μs were recorded during 1 ms test time of our HST4 facility. Measured time-resolved evolution of shock structure around the model is compared with Schlieren technique and CFD simulation results to validate the proposed technique. Infinite fringe interferograms are evaluated using active contour technique and Fourier transform fringe analysis to extract density data of the hypersonic flow field around the model. Estimated time-resolved density data show the variation in the density within shock layer.

Keywords: time-resolved interferometry, digital holography, flow visualization, hypersonic flow

1 Introduction
Holography is a powerful non-destructive tool for investigating variety of problems in physics and engineering. One such engineering application of holography is in flow analysis. Holographic interferometry has been successfully used on a range of shock tunnels and expansion tubes to study a variety of problems related to high-speed flows.1–7 Both qualitative visualization and quantitative measurements are made on complex shock wave configurations utilizing both finite-fringe and infinite-fringe interferometry. Apart from visualization of the flow field, analysis of without-flow and with-flow interferograms can provide information about the density distribution, ionization levels and temperatures throughout the flow field. In conventional holography applied to short duration hypersonic shock tunnels, recording of the interferogram during a tunnel run is done in the following way: typically a high power pulsed laser (Nd:YAG or Ruby) timed to produce light pulse during the test time of the facility, is used as the light source. Just prior to firing the tunnel the laser is pulsed recording a no-flow hologram. A second hologram with flow is recorded during the test (steady flow) time of the facility. The hologram has to be developed in a dark room. Reconstruction will be performed using another laser, then the hologram will be digitized with a CCD/C莫斯 camera before transferring it to a computer for subsequent fringe analysis. The main disadvantages of this method of hologram recording in high speed flow studies are: i) necessity to use a bulky pulsed laser system, ii) requirement of high quality optics, extremely stable and dark environment and, iii) inability to record time-resolved (multiple) holograms. Importance of multiple high speed imaging in hypersonic impulse facilities, existing optical diagnostic techniques to achieve such measurements, their complexities, limitations and future directions are discussed in ref.8 Recent applications of digital holographic interferometry (DHI) in diverse fields such as microscopy,9 3-D imaging,10 complete MEMS characterization,11,12 particle image velocimetry13 and in biology14,15 etc. have demonstrated the usefulness of this method and offers new perspectives in color DHI using three wavelengths.16 Recently digital color holography is applied to flow analysis and structural mechanics studies.17–19 In digital holography, the hologram which is the
interference of object and reference waves is recorded in digital form using CCD or CMOS sensors and then numerically reconstructed in the computer. The main advantage of DH is: it allows real time recording of interferograms directly into a digital camera. For high enthalpy, high speed flow facilities having short test duration this approach coupled with high-speed camera opens the possibility for multiple interferogram images in a single shot, which is not possible with conventional approach.

There are some reports in the literature on the application of digital holographic interferometry in flow studies. Carlos Pérez-López et al\textsuperscript{20} have reported the use of a very high speed camera in digital holographic interferometry using cw laser to record an out of plane sensitivity setup on a latex balloon. Aguyo et al\textsuperscript{21} have demonstrated use of high speed DHI in insect wing deformation studies. Whole-field noncontact measurement of displacement, velocity, and acceleration of a vibrating object using high speed digital holography is also reported by Fu. et al.\textsuperscript{22} Application of DHI to study the wind tunnel flows where the typical test time (steady flow) is in few (5–10) seconds, is also reported.\textsuperscript{7} Most of the reported DHI experiments in flow studies are either at low speeds (supersonic) or the systems having the longer test times (10s of seconds). However, application of DHI technique using very high speed camera in short duration high enthalpy hypersonic flow facilities where the flow speed is Mach 6 and above, is not explored yet.

In this manuscript, we report on the application of digital interferometry technique integrated with short duration hypersonic shock tunnel facility (HST4) having 1 ms test time, for quantitative time-resolved flow visualization. Proposed technique is developed using the combination of continuous wave (cw) laser, Mach-Zehnder interferometer (MZI) and very high-speed (0.2 million fps) CMOS digital camera. A 40° blunt cone model with a base radius of 51 mm flying at Mach 6 has been used to demonstrate the proposed technique. Results of the high speed digital interferometric flow visualization are compared with that of Schlieren flow visualization and computational fluid dynamics (CFD) studies to validate the proposed technique. Digital interferograms are analyzed using improved active contour technique and Fourier transform fringe analysis to extract density information of the flow field around the test model. The proposed technique holds promise for the time-resolved flow dynamics studies in hypersonic test facilities.

2 Experiment

The experiments are carried out in the HST4 hypersonic shock tunnel, which consists of a stainless steel shock tube of 50 mm internal diameter connected to a divergent conical nozzle of 300 mm exit diameter and dump tank. The driver and driven sections of the shock tube are separated by an aluminium diaphragm. Three pressure sensors are mounted along the length of the shock tube at pre-determined locations for the measurement of shock speed and the stagnation pressure at the entry to the nozzle. The Mach 6 hypersonic flow from the nozzle goes through a 450 mm long test section of 300 mm \( \times \) 300 mm square cross section. The model was mounted across the hypersonic flow along with the Pitot probe in the test section. Consistency of all the results with and without flow was ensured by repeating the experiments and the average values of the measured parameters and the best holographic images are presented in this paper.

A steady state Mach 6 hypersonic flow exists only for about 1 ms in this facility, during which the dynamics of hypersonic flow has to be monitored. The specific free stream enthalpy of the flow in the test section of shock tunnel can be varied by choosing the metal diaphragm of appropriate thickness. Other details of the HST4 facility can be found in ref.\textsuperscript{23} Current set of experiments are performed using helium as the driver gas and air as the test gas respectively to achieve required stagnation condition. The typical free stream conditions for stagnation enthalpy of 1.99 MJ/kg are given in the Table 1. The layout of optical arrangement used is shown in Fig. 1a. and 1b. shows the photograph of HST4. A diode pumped Nd:YAG cw laser (Spectra Physics), emitting 532 nm wavelength and a power of 120 mW is used to create interferogram. A Phantom 7.2 high speed digital camera (Ms. Vision Research, USA) was used to record the interferogram images. Though the camera can record speeds up to 0.2 million fps with high spatial resolution dependent on recording speed, for this set of experiments we have optimized the speed at 8350 fps and exposure time of 60 \( \mu \)s to get best possible resolution. The laser beam is split into two beams, one passing through test section as object (imaging) beam and the other reference beam. Laser beam in the test section is expanded to fill the test section windows (6 inches dia), passed through the test section covering the region of interest around the model before being imaged onto the CMOS camera. The reference laser beam is passed over the test section, expanded, and then sent to the camera to form the interference. By conducting
a series experiments a best combination of laser power, camera speed (fps), exposure time and time resolution has been arrived to achieve good DH images. Flow field around the model in the test section is also visualized using the Schlieren visualization technique for comparison. Schlieren images of the flow field are recorded using the same (Vision Res. Inc. USA) high speed camera at the acquisition rate of 10000 frames per second in the current set of experiments.

### 3 Results and Discussion

Conventional holography experiments in high speed flow facilities are normally performed by recording holograms of without-flow and with-flow situations on a holographic plate/film. After chemical processing of the film, reconstruction of the hologram generates the original beam that is allowed to overlap and forms the interferogram. This image is digitized using a CCD camera then transferred to a computer for fringe analysis. In the present approach the interferogram is directly recorded into the digital camera. After careful optical alignments, fringe pattern was imaged onto

![Figure 1a](Image1) Optical schematic of the digital interferometric visualization set-up used in HST4 facility.

![Figure 1b](Image2) Photograph of hypersonic test facility (HST4) used for this experiment.

### Table 1: Typical test conditions used in HST4 facility at $M_\infty = 6$.  

<table>
<thead>
<tr>
<th>Test Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driver gas</td>
<td>Helium</td>
</tr>
<tr>
<td>Primary diaphragm thickness (mm)</td>
<td>1.5</td>
</tr>
<tr>
<td>Shock Mach number, $M_\delta$</td>
<td>3.63</td>
</tr>
<tr>
<td>Stagnation pressure, $P_o$ (kPa)</td>
<td>1250</td>
</tr>
<tr>
<td>Stagnation enthalpy, $H_o$ (MJ/kg)</td>
<td>1.99</td>
</tr>
<tr>
<td>Free stream Mach number $M_\infty$</td>
<td>6.0</td>
</tr>
<tr>
<td>Free stream static pressure, $P_\infty$ (Pa)</td>
<td>1027</td>
</tr>
<tr>
<td>Free stream static temperature, $T_\infty$ (K)</td>
<td>260.1</td>
</tr>
<tr>
<td>Free stream static density, $p_\infty$ (kg/m$^3$)</td>
<td>0.0137</td>
</tr>
<tr>
<td>Free stream velocity, $V_\infty$ (m/s)</td>
<td>1858</td>
</tr>
<tr>
<td>Free stream unit Reynolds number, $R_e$ (m$^{-1}$)</td>
<td>$1.54 \times 10^6$</td>
</tr>
</tbody>
</table>
the high speed camera. Then the interferograms are recorded one before the flow and many interferograms during the flow. Operation of the camera was synchronized with the shock tunnel flow using a trigger pulse generated by the pressure sensor located at the pre-determined location near the test section of the shock tube. Using Pitot pressure measurement it was estimated that the presence of a Mach 6 steady flow for about 1 ms, which in turn is taken as the useful test duration of the facility. Several runs are taken and typical interferometric images of the steady state flow of one of the successful runs are shown in Fig. 2. The flow field image around the model recorded by Schlieren visualization technique in a separate experiment, under same flow conditions is also shown in the same figure for comparison. The shock location and the shape observed by digital interferometric method match well with that of Schlieren visualization image. However, the interferometric image is magnified by about 2x because of the lenses used. Fig. 3 shows the sequence of time-resolved interferometric images of the hypersonic flow field around the blunt cone model recorded during the flow starting time up to flow termination. The same sequence of flow dynamics under same flow conditions, is recorded using Schlieren visualization technique and other details are reported in ref.24. The time evolution for the flow filed starting from tunnel starting, steady state flow and the flow termination observed by DI technique is in good agreement with the well-established Schlieren images. The nozzle starting time, steady flow and flow breakdown shown in the sequential interferometric images are confirmed with the simultaneously measured Pitot pressure.

Shock stand-off distance is one of the important and preliminary measurements generally done in high speed flow visualization, and it depends on the body geometry and the flow Mach number. Normally most of the blunt-body hypersonic flow fields are characterized by the shock stand-off distances ahead of the blunt body. Shock stand-off distance measured at the centre of the model surface by digital interferometric method is 2.12 mm whereas the same measured by Schlieren technique is 2.08 mm. Shock stand-off distance along the length of the blunt cone surface is digitally measured from the recorded interferogram. The hypersonic flow around the blunt cone was also simulated by solving the Navier–Stokes equation in the axisymmetric formulation using commercial CFD (Computational Fluid Dynamics) package CFX 5.7. The experimentally visualized bow shock wave in front of the 40° blunt cone model along with the results from the computational study is shown in Fig. 4 for comparison. Measured shock stand-off distances along the surface of the blunt cone obtained both from interferometry and schlieren experiments, and CFD studies are plotted in Figure 5. The results obtained are matching well within ±5% error compared to Schlieren and CFD results as is clearly seen in Figs. 4 and 5. These results clearly show that using the proposed time-resolved digital interferometric visualization technique it is possible to study the complex flow field dynamics in hypersonic shock tunnels.

The main purpose of using holographic interferometry in high speed flow studies is to extract density information in the flow field around the body. In the present approach of digital interferometry technique described above, the infinite fringe interferograms are produced by keeping the optics fixed both for reference and object beam between the exposures (with...
and without flow) and recorded directly using the digital camera. The in-line hologram is the interference of object wave with the in-line reference wave at the CCD plane \((\xi, \eta)\), can be represented in general form as:

\[
H(x, y) = |O(x, y) + R(x, y)|^2 = |O(x, y)|^2 + |R(x, y)|^2 + O^*(x, y)R(x, y) + O(x, y)R^*(x, y)
\]

\(1\)

Figure 3: Sequence of time-resolved interferometric images of the hypersonic flow field around the blunt cone model with a time resolution of 139\(\mu\)s.

Figure 4: Comparison of the shock wave structure in front of the blunt cone a) interferometric and Schlieren images (lower half) and b) interferometric and CFD (lower half) results.
where \( x \) and \( y \) are the spatial pixel coordinates, \( O(x, y) \) and \( R(x, y) \) are the reference and object beam amplitudes, and * denotes the complex conjugate amplitude of these quantities.

If the CCD contains \( M \times N \) pixels with pixel size \( \Delta x \times \Delta y \), then the digitally sampled hologram can be written as,

\[
H(m,n) = \left[ H(m,n) \otimes \text{rect} \left( \frac{\xi}{M \Delta \xi}, \frac{\eta}{N \Delta \eta} \right) \right] 
\times \text{rect} \left( \frac{\xi}{M \Delta \xi}, \frac{\eta}{N \Delta \eta} \right) \text{comb} \left( \frac{\xi}{\Delta \xi}, \frac{\eta}{\Delta \eta} \right)
\]

(2)

where \( \otimes \) represents the two-dimensional convolution and \( (\alpha, \beta) \in [0,1] \) are the fill factors of the CCD pixels.

The reconstruction of hologram is a diffraction process. When the same reference wave is used to illuminate the hologram, the wave field reconstructed at the distance \( d \) at the image plane \( (x', y') \) is obtained as,

\[
U(x', y') = \frac{\epsilon}{i \lambda d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(\xi, \eta) R(\xi, \eta) e^{i \frac{2\pi}{\lambda d} \left[ (x' - \xi)^2 + (y' - \eta)^2 \right]} d\xi d\eta
\]

(3)

To obtain the relative phase map between the two flow states a Fourier transform and its inverse is applied to each digital holographic interferogram, and upon subtraction of the two inverse transformed images, the resulting relative phase can be found from,

\[
\Delta \phi_n = \arctan \left[ \frac{\text{Re}(I_{n-1}) \text{Im}(I_n) - \text{Im}(I_{n-1}) \text{Re}(I_n)}{\text{Re}(I_n) \text{Im}(I_{n-1}) - \text{Im}(I_n) \text{Re}(I_{n-1})} \right]
\]

(4)

where \( \Delta \phi_n \) is the relative wrapped phase map between a reference (without flow) state hologram \( (I_{n-1}) \) and \( n \)-th (with flow) hologram \( (I_n) \). Re and Im represent the real and imaginary part of a complex number.

Several fringe analysis techniques have been proposed in literature depending on the experimental and object conditions, and also on application of DHI.25–28 By extracting the phase information from the fringe pattern a quantitative height map can be obtained. The height map may represent a surface profile, density profile, a displacement, or the optical path difference through a phase object. However, using our experimental results obtaining this height map was not straightforward due to ambiguities in the fringe pattern, discontinuous or non-uniform phase jumps, object size, speckle noise (as seen in Fig. 2b) and also on the resolution limitations of the imaging system. Especially high speed flow studies using DHI in short duration test facilities where the flow is over bigger objects (test models), it is difficult to achieve uniformity in the fringe patterns because of high pressure diaphragm bursting experimental conditions. This makes...
holographic studies of high speed flows involving bigger objects (or larger field of view) most often to rely on non-uniform finite fringe patterns. Therefore in the present analysis we have adopted a different methodology for the evaluation of non-uniform fringe interferograms of bigger objects. First the problem of identifying the relevant phase data in Fourier domain of the interferogram is reduced to an object detection problem by normalizing the logarithm of the Fourier domain image. The active contours technique is used to carry out the object detection with an iterative process.

Fringe analysis is carried out following the steps as described in refs. However, we adapted different methods in few steps in ref. to extract phase information from our experimental results. As a first step, object edge detection was carried out to have clear separation of the flow field from the model surface. An improved active contour technique is used for edge detection and to recover the phase data from non-uniform fringe patterns for phase unwrapping. Details of the mathematical steps involved are given in ref. Fig. 6 shows the typical phase wrapped and unwrapped images of one of the steady interferometric images (from Fig. 3) of blunt cone missile model subjected to Mach 6 flow using active contour and Fourier transform fringe analysis techniques.

Density determination from the unwrapped phase is done considering the two dimensional flow. In case of two dimensional flows, the phase is expressed as,

$$\phi(x, y) = \frac{2\pi}{\lambda} [\eta_{\text{ref}} - \eta_{\text{flow}}(x, y)] W$$  

where $W$ is the distance that the light travels through the shock tunnel (phase-shifting medium), $\lambda$ is wavelength of the light used, and $\eta_{\text{ref}}$ is the refractive index of the reference (without flow) and $\eta_{\text{flow}}$ is the refractive index with flow. For a perfect gas of uniform composition, as discussed by Merzkirch, the refractive index $\eta$ can be directly related to the density $\rho$ of the gas and its Gladstone-Dale coefficient $K$, so that it can be written as,

$$\eta = 1 + K \rho$$  

According to Liepmann and Rohko the Gladstone-Dale coefficient can be expressed in terms of its value at the standard density $\rho_s$, then the above equation becomes:

$$\eta = 1 + \frac{\rho - \rho_s}{\rho_s}$$  

For the gas used in the present work, values of $\beta$ and $\rho_s$ are taken from Liepmann and Rohko, and free stream static density was taken as $0.0137$ kg/m$^3$. Using the above equations, the density of the flow is determined from:

$$\rho_{\text{flow}} - \rho_{\text{ref}} = \frac{\rho - \rho_s}{2\pi K_0 V} [\phi_{\text{ref}} - \phi_{\text{flow}}]$$  

By processing the subsequent interferograms recorded during the stagnated flow duration (1 ms) we have estimated dynamic density profile. The time-resolved density profile of the flow field around the model obtained from the fringe analysis for three interferometric (Fig. 3 steady state flow) images are shown Fig. 7(a), (b), (c). It is observed that there is a slight density fluctuation during the steady flow. These variations are associated with the changes in pressure temperature and the chemical reactions within the bow shock region. Thus, quantitative estimation of density variation in high speed flows is demonstrated through time-resolved digital interferometry technique. Understanding of these density fluctuations around the obstacle, particularly the boundary layer is of great importance in hypersonic flow studies. Density profile of the flow field generated using CFD simulation is shown in Fig. 7(d) for comparison.

It is observed that there is a fairly good comparison between the experimental and CFD profiles at the tip of the cone in qualitative sense. However, quantitatively there is a discrepancy between CFD and experimental results at other locations around the model. These discrepancies are because of the slight differences in the initial conditions used by CFD and the actual experimental conditions during the short (1 ms)
test time. One possible cause for the difference in initial conditions is that the model might have moved soon after the hypersonic flow struck on the model, causing the angle of incidence between object and shock thereby affecting the initial conditions of the shock tunnel and the flow dynamics. This can be seen in the images in Fig. 2. The experimental model appears to be slightly non-symmetric. This has led the experimental configuration and the conditions to develop into slightly different from that predicted by CFD. Second possibility is the vibration, caused by bursting of the diaphragm, leading to system vibrations and hence resulting in non-uniform fringe pattern. One more possibility for non-uniform fringe patterns may be due to the aberrations caused by the thick (1.5 inch) glass test windows of the shock tunnel. These proposed explanations need to be further investigated in future work to improve the quality of the fringe patterns, and to reduce the uncertainties with respect to initial conditions used both in the experiment and CFD simulation. Also, as seen in Fig. 7 the resolution of density profile is not of high quality. However, the resolution of the density distribution in the flow field can be improved further by using high speed CCD camera having smaller pixel size and higher number of pixels. Also the fringe analysis method has to be improved to get best possible information out of it. All these issues are planned to be addressed in our future experiments. Thus, we have developed and successfully used the digital interferometry technique for quantitative high speed flow visualization in short duration hypersonic shock tunnel. By processing the sequence of interferograms recorded during the steady state flow condition, the dynamic density fluctuation during the high speed flows can be studied. Further analysis of these density variations will be very useful in understanding the other processes in compressible flows and shock wave focusing.

4 Conclusion

In conclusion, time-resolved digital interferometric flow visualization technique integrated with hypersonic shock tunnel is developed. Time-resolved digital interferograms of Mach 6 hypersonic flow have been successfully recorded using a simple combination of cw laser, MZ interferometer and high-speed digital camera. Using the proposed laser based DHI visualization facility it is now possible to capture the dynamic digital interferogram of hypersonic flow in the shock tunnel where the typical test time is 1 ms. Visualization results of the proposed technique are in good agreement with that of Schlieren technique and CFD results. Further, the digital interferograms are analyzed to extract the time-resolved quantitative density distribution of the hypersonic flow field around the model. The experimental results have been compared...
with CFD simulation and the discrepancies are attributed to errors in initial conditions, and also the non-uniform fringe distribution and hence the phase. The proposed unique combination of the instruments and the developed technique hold promise for the future investigation of other processes in compressible flows such as density distribution, time evolution of the density and shock wave focusing.

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References


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