Two-temperature noble gas plasmas

Part 1: Thermodynamics and transport coefficients
Part 2: Transport properties of argon plasma

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Received on January 21, 1983; Revised on May 3, 1983.

Abstract

From thermodynamic principles, species, kinetic temperature and state equations are defined. The different equilibrium models to compute a two-temperature plasma composition are presented. The effect of lowering of ionization potential on plasma composition is found to be very small. Expressions for two-temperature plasma transport coefficients are presented. Using the expressions for electron and heavy particle transport coefficients, the properties of non-equilibrium argon plasma are computed and the computed values in the limit $\theta = 1$ agree satisfactorily with the exact theory.

Key words: Local thermodynamic equilibrium, two-temperature plasma, ambipolar and reactive heat diffusion coefficients, electrons, heavy-particles, transport properties.

I. Introduction

In recent years, a great deal of attention has been paid to studies of high temperature non-equilibrium plasma situations such as glow discharges, constricted arcs, plasma jets, shock-heated plasmas and magneto-gasdynamic energy conversion devices. In the above cases, the non-equilibrium effects observed are ionization non-equilibrium, non-equivalent of translational energy between electrons and heavy particles and chemical non-equilibrium depending on the physical situation. For example, in a constricted arc, the processes which tend to promote these non-equilibrium effects are external electric field, thermal conduction, ambipolar diffusion and elastic/inelastic collisions. An effective utilization and exploitation of the plasma under such situations requires a knowledge of the thermodynamics of a plasma state first and subse-
quently the study of transport processes based on a proper kinetic theory yields transport coefficients.

Only those references related to the study of thermodynamics and transport phenomena of a two-temperature plasma are cited in the present work since the literature available on this subject is wide and a vast growing one. The method of approach of the study of thermodynamics of two-temperature plasma follows that of Drawin, Pfenecr and Bose7. Devote8–10 has presented expressions for transport coefficients based on a rigorous kinetic theory for an argon plasma under equilibrium conditions. Kannappan and Bose11,12 have reviewed several methods for calculation of transport properties of a plasma under equilibrium and non-equilibrium conditions. Two-temperature plasma composition and transport properties were presented by them for argon and helium plasmas for electron to heavy particle temperature ratio of three.

The highlights of the present paper are as follows: 1. Two different equilibrium composition models based on the Saha equation are presented. The partial derivatives of electron mole fraction with respect to electron and heavy temperatures are discussed for the two models. 2. The concept of ambipolar diffusion is extended to two-temperature plasma situations. 3. Equations are presented to compute heat due to conduction and diffusion-reaction.

Part I deals with the definitions of local thermodynamic equilibrium and kinetic temperatures of species. Multiple temperature concept is introduced in context with non-equipartition of energy. Thermodynamic state equations and the Saha equation are described/discussed in computing the plasma composition. Also shown in this section is the derivation of transport coefficients for a two-temperature plasma. Simplified expressions for the transport properties of electron and heavy subgases constituting an ionized gas mixture obtained from a rigorous kinetic theory are presented in Part 2. The computed transport properties for argon at \( P = 1 \) bar are presented followed by a brief discussion on the variation of species' transport properties with temperature.

1.1. Local thermodynamic equilibrium

In order to allow a laboratory plasma system to have thermal and chemical variations and still be characterized by a unique temperature and pressure at a point, the concept of local thermodynamic equilibrium originated. Under the restrictions of LTE, the state of a system at a point is defined uniquely by a definite set of thermodynamic functions in conjunction with the equilibrium law of mass action. However, the state parameters are allowed to vary spatially.

Depending upon the purpose of study, the plasma system conditions and on the definition of temperature, numerous definitions of LTE are possible. For more details, the reader is referred to the works of Pfenecr13 and Drawin.6. It is seen that the correct appropriate interpretation depends upon which modes of energy storage are dominant...
for a plasma under given conditions. In the present work of collision-dominated three-component plasma, the dominant modes of energy storage are the species, thermal energy and the ionization energy. Hence LTE exists when $T_e = T_i = T_n = T_t$ at each point in the plasma where each $T$ is defined via the appropriate equilibrium equation.

12. Non-equipartition of energy and multiple temperature concepts

In a constricted arc, Incropera and Viegas\(^2\) made a time-scale study in which a representative relaxation time was associated with each of the physical mechanisms identified to promote non-equilibrium. Non-equipartition can be studied with reference to elastic collisions in a gas mixture containing species of disparate mass. McDaniel\(^8\) has shown that, on the average, the fraction of the initial kinetic energy possessed by species with mass, $m_i$, and transferred to a less energetic species with mass, $m_j$, during a single elastic collision. It is given by:

$$\Delta E = \frac{2m_i m_j}{(m_i + m_j)^2} \left( m_i + m_j \right)$$

If $m_i = m_j$, $\Delta E = 1/2$.

For an argon atom $m_i = m_e = 6.635 \times 10^{-26}$ kg and for an electron, $m_e = 9.1 \times 10^{-31}$ kg, the energy exchange is $\Delta E = 3 \times 10^{-5}$.

Thus it is apparent that energy transfer during an elastic collision between species of equal mass is highly efficient whereas it is highly inefficient between species of disparate mass.

Consider now the perturbation due to applied electric field, on a partially-ionized plasma at a certain instant in space. The electrons will very quickly equilibrate among themselves and approach an equilibrium (Maxwellian) thermal energy distribution, then heavier particles equilibrate among themselves and approach Maxwellian distribution. Under this condition of non-equipartition of energy, two distinct equilibrium distributions of thermal energies corresponding to lighter (electrons) and heavy particles are established. Hence the mixture of species of disparate mass is described by two different kinetic temperatures $T_e$ and $T_n$.

From an energy balance at steady-state condition\(^7\), the departure of the electron temperature from the gas temperature is given by the following expression

$$T_e = T_n + \frac{2m_h}{3k_B \delta} \left( \frac{f}{e n_e} \right)^2$$

13. Thermodynamic state equations and composition of a two-temperature plasma

The following are assumptions, approximations, definitions and derived relations:
(i) Assumptions
1. The argon plasma is a three-component mixture of free electrons, ground state singly charged ions and atoms or neutrals.
2. All species thermal velocity distributions are Maxwellian with appropriate kinetic temperatures. Atoms and ions have the same velocity or atom and ion temperatures are equal ($T_a$).
3. The plasma is quasi-neutral with equal electron and ion number densities.
4. All particle collisions are elastic binary collisions.
5. All radiation is optically thin, hence the influence of radiation term is not included in the present analysis.
6. Energy storage in the form of internal electronic excitation is negligible.

(ii) Thermodynamic state equations of a two-temperature plasma

Equation of state

$$p = n_e k_B T_e + k_B T_h \sum_{j \in H} n_j$$  \hspace{1cm} (3)

$$= nk_B T_e \left[ x_e (0 - 1) + 1 \right] / 0$$, \hspace{0.5cm} where \hspace{0.5cm} 0 = T_e / T_h

$j \in H$ means that species, $j$ corresponds to a set of heavy particles, $H$.

Caloric equations of state of species

$$h_j = \frac{5}{2} k_B T_j$$  \hspace{1cm} (4)

Equation (4) defines actually species translation enthalpy. However, in the calculation of heavy species enthalpy, the energy residing in the plasma in the form of electronic excitation, though small, is taken into account. Accordingly, the species total enthalpy is

$$h_j = \frac{5}{2} R^* T_h + h_{j, exec} + h_{j, o, i \in H}$$  \hspace{1cm} (5)

where

$$h_{j, exec} = R^* T_e \frac{\delta \ln Z_{j, exec}}{\delta T_e}$$

$$Z_{j, exec} = \sum_{j \in H} g_{jk} \exp \left[ - E_{jk} / k_B T_e \right]$$

$$h_{j, o} = \text{heat of formation at } 0^\circ K$$

Enthalpy of mixture is

$$h = \frac{5}{2} R^* \left[ x_e T_e + (1 - x_e) T_h \right] + \sum_{j \in H} x_j h_j$$

$$+ R^* \sum_{j \in H} x_j \left( \frac{g_{jk} E_{jk} / k_B}{g_{jk} \exp \left( - E_{jk} / k_B T_e \right)} \right)$$  \hspace{1cm} (6)
Approximations and definitions

\[ m_e/m_1 \ll 1, \quad n_i \approx m_i, \quad n_e \approx n_i, \]

\[ \alpha = n_e/(n_e + n_n), \quad R^* = k_B N_A, \quad n_i = \frac{R^*}{l_B} M_i, \]

\[ x_j = n_j/n, \quad \rho = \sum_i n_i M_i = \frac{k_B}{R^*} n \left[ (m_e/m_n) + (n_e + n_n) m_n \right]. \tag{7} \]

Plasma composition and discussion of models A and B

Numerical computation of transport properties of coefficients requires an accurate evaluation of plasma composition for which two different equilibrium composition models are used.

Following the principle of minimum chemical potential, Monti and Napolitano\textsuperscript{10} and Veisi\textsuperscript{9,11} has given the following expression for a three-component plasma in a state of chemical, not thermal, equilibrium, which we refer to as model A.

\[ S = n x_i \left( \frac{x_i-1}{x_i} \right)^{1/3} \tag{8} \]

From the basics of kinetics and use of the product of partition function evaluated at electron temperature \( T_e \), Kerrebrock\textsuperscript{21} presented the following expression model B.

\[ S = n \left( \frac{x_e x_i}{x_i} \right) \tag{9} \]

where the subscript \( i = 1 \) for neutral particles and \( i = 2 \) for singly charged ions.

The Saha function, \( S_i \), is given by

\[ S_i = 2 \frac{Z_i-1}{Z_i} \left[ \frac{2 \pi m e k_B T_e}{\hbar^2} \right]^{1/2} \exp \left[ \frac{(I_i - \Delta I_i)/k_B T_e)}{\gamma} \right] \tag{10} \]

In eqn. (9), \( I_i \) is the ionization potential and \( \Delta I_i \) takes into account the lowering of the ionization potential due to fields of the charged particles. The expression for \( \Delta I_i \) for the present case is as follows:

\[ \Delta I_i \approx \frac{i^2}{2} \sqrt{n_e/(0 + 1)/T_e} \tag{11} \]

It is found that \( \Delta I_i \) is very small even if \( T_e \) is small, for example, near a cold wall. Consideration of \( \Delta I_i \) affects the mole fraction only very marginally\textsuperscript{22}. The partition functions \( Z_{11}, Z_i \) are evaluated using the Unsöld criterion\textsuperscript{23} for cutting off the series.

From the following definitions

\[ x_i + \sum_{ii} x_i = 1 \tag{12} \]
a generalized form of the Saha equation for model \( A \) in the present case is

\[
x_{i+1}^\theta + ix_{i+1}^\theta + (i + 2) S_i^\theta x_{i+1} - S_i^\theta = 0
\]

Figures 1 to 4 present representative results of calculation of argon species mole fractions at one atmosphere of pressure and for \( \theta \) values up to 10 for both models \( A \) and \( B \).
From figs. 1 and 2 it is seen that the variation of $x_e$ with $T_e$ for $\theta > 1$ is generally much larger in model $A$ than in model $B$.

Table I shows the values of electron mole fraction for various values of $T_e$ and $\theta$. Around $15000^\circ K$ and at one atmosphere the number densities of equilibrium species
in an argon plasma are almost equal. However, the trend of variation of \( x_e \) in model A in the range 5000 to 15000° K is different from that above 15000° K.

In the expressions for species, diffusion flux and heat flux shown in the next section, \( \nabla x_e \) appears and it is also noted that \( x_e \) is a function of electron and heavy particle temperatures.

Accordingly, the expression of \( \nabla x_e \) is

\[
\nabla x_e = \frac{\partial x_e}{\partial T_h} \frac{dT_h}{dy} + \frac{\partial x_e}{\partial T_e} \frac{dT_e}{dy}
\]

(15)

Closed form analytical expressions for partial derivatives of \( x_e \) with respect to \( T_e \) and \( T_h \) are given in Appendix I for both models A and B.
In view of the peculiarity of variation in $x_e$ in model $A$, we attempt to study and interpret the partial derivatives with physical relevance to our study. With the aim of understanding the situation further, $x_e$ is plotted in Fig. 3 for both the models as a function of $T_e$ with $T_h$ as parameter and in Fig. 4 as a function of $T_h$ with $T_e$ as parameter. It is seen from Fig. 3 that $\partial x_e/\partial T_e$ is positive at all $T_e$ and $T_h$ for both the models while $\partial x_e/\partial T_h$ can be seen in Fig. 4 to be slightly negative for model $A$ at $T_e \approx 16000$ K. These can also be verified numerically from the expressions provided in Appendix I.

It is also well known and is the basis of electron temperature measurement with the help of electrostatic probes, that $T_e$ does not change significantly near the cooled wall, whereas $T_h$ changes from wall temperature to the freestream temperature. Therefore, at $T_e > 15,500$ K, atleast theoretically for model $A$ to be valid, there is to be a small electron flux and resultant heat flux by diffusion from lower to higher temperature. However, model $A$ violates the second law of thermodynamics in this range of electron temperatures and also the total heat flux due to pure conduction and diffusion becomes lower than due to pure conduction alone. It is also found for the absolute magnitude of this reactive heat flux for both the models is much smaller in comparison with the pure conductivity values. In view of the above reasoning it is suggested to put $\partial x_e/\partial T_h = 0$ for model $A$ at this temperature range where it is negative.

Table I

<table>
<thead>
<tr>
<th>$T_h$ K</th>
<th>$x_e$ ($\theta = 1$)</th>
<th>$x_e$ ($\theta = 5$)</th>
<th>$x_e$ ($\theta = 10$)</th>
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<td>0.000402</td>
<td>0.000105</td>
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<td>0.1072</td>
<td>0.0078</td>
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<td>0.2711</td>
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<tr>
<td>3500</td>
<td>0.4979</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Species diffusion velocities and current densities

With the knowledge of the equilibrium chemical composition of argon plasma, we need to study the species diffusion velocities and current densities due to concentra-
tion gradients and applied electric field under thermal non-equilibrium conditions. This follows from classical approach\(^{24, 25}\).

It is assumed that in any given state \((p, T_e, T_h)\) only electrons, \(i\) and \((i + 1)\)th ions are present in the gas. From eqns. (12) and (13), \(x_i\) and \(x_{i+1}\) are expressed in terms of \(x_e\) as

\[
\begin{align*}
    x_i &= i - (i + 1) x_e \\
    x_{i+1} &= 1 + i (x_e - 1)
\end{align*}
\]
Upon operating with del operator, eqns. (16) and (17) are written as

$$\nabla x_i = -(1 + i) \nabla x_e$$

$$\nabla x_{i+1} = i \nabla x_e$$

(18)

Current densities of species in a plasma with impressed electric field and concentration gradients are as follows:

$$J_i = -en_i \nabla \rho_i = en_i b_i E + eD_i \nabla x_i$$

$$J_i = (i - 1) en_i V_i = e(i - 1) n_i b_i E - e(i - 1) nD_i \nabla x_i$$

$$J_{i+1} = ien_{i+1} \nabla \rho_{i+1} = ein_{i+1} b_{i+1} E - einD_{i+1} \nabla x_{i+1}$$

(19 a-c)

Substituting the gradients of mole fraction of individual species given in eqns. (18) and (19) into (19 a-c) current densities of species $i$ and $(i + 1)$ can be written in terms of $\nabla x_e$. The expressions for the species diffusive velocities in the absence of applied field, $E$, obtained from eqns. (19 a-c), are expressed in terms of $\nabla x_e$:

$$V_i = -(D_i/x_i) \nabla x_e$$

$$V_i = -(1 + i) (D_i/x_i) \nabla x_e$$

$$V_{i+1} = + i (D_{i+1}/x_{i+1}) \nabla x_e$$

(20 a-c)

It may be observed that eqns. (20 a-c) satisfy the condition, $\Sigma \rho_j V_j = 0$.

15. Ambipolar diffusion coefficient

When diffusion is considered, one of the two possible approaches is used to calculate diffusion flux. One approach involves the use of individual species momentum equations which are solved to give separate species diffusion velocities as well as resultant electric field intensity due to space charge set up by charge separation. A second approach involves invoking the quasi-neutral assumption, thereby equal electron and ion diffusion velocities and an appropriate ambipolar diffusion model is used. In the present study, the later approach is employed by neglecting thermal diffusion.

From eqn. (19 a-c), we get

$$J_e = \frac{ix_e b_e D_{e}}{x_e b_e + x_{i+1} b_i D_{i+1}} \nabla n_e$$

$$\approx \frac{ix_e b_e D_{i+1} + x_{i+1} b_i D_i}{x_e b_e} D_e n \nabla x_e$$

$$= n \left( iD_{i+1} + x_{i+1} b_{i+1} D_e \right) \nabla x_e$$

(21)

Noting that

$$\frac{b_{i+1}}{D_{i+1}} = \frac{i_e}{k_B T_h} ; \quad \frac{b_e}{D_e} = \frac{e}{k_B T_e}$$

$$\frac{b_{i+1}}{D_{i+1}} = \frac{i_e}{k_B T_h} ; \quad \frac{b_e}{D_e} = \frac{e}{k_B T_e}$$
We get
\[ \frac{J_e}{en} = iD_{i+1} \left( 1 + \theta \frac{x_{i+1}}{x_i} \right) \nabla x_e. \]

Hence, the ambipolar diffusion coefficient for a two-temperature plasma is given by
\[ D_{\text{amb}} = i \left( 1 + \theta \frac{x_{i+1}}{x_i} \right) D_{i+1}. \]

Equation (23) is a relation for an 'effective diffusion coefficient' and is a function of ratio of electron to heavy temperature, mole fraction and diffusion coefficient of species. This relation also holds good even in the absence of applied field, \( E \). For \( \theta = 1 \) and \( i = 1 \), eqn. (23) reduces to the familiar expression for diffusion under equilibrium conditions. The numerical results and discussion on ambipolar diffusion are given in Part 2.

1.6. Reactive heat conductivities and heat fluxes of species

For a reacting and radiating gas mixture under certain circumstances, two components of energy transfer mechanisms which give rise to an effective increase in the value of the thermal conductivity coefficient, were identified. They are diffusion and radiation processes. A total thermal conductivity coefficient of \( j \)-th species in a mixture is defined as
\[ k_j = k_{ej} + k_{pj} + k_R \]

If \( V_j^f \) is the relative velocity due to diffusion of \( j \)-th species carrying an enthalpy, \( h \), the total enthalpy flux due to diffusion carried on to the colder region in a two-temperature plasma, is given by
\[ H = \sum \rho_j V_j^f h_j, \quad j = e, i \text{ and } n \]

Since \( \rho_j = (k_B/R^*) m_p n_j = m_v n_j/N_A \) and \( H_j = m_p h_j \) we can arrive at the following expression for enthalpy flux and using eqns. (20) and (23)
\[ H = -\frac{n D_{\text{amb}}}{N_A} (H_e + H_{i+1} - H_i) \frac{\partial x_e}{\partial T_e}. \]

Upon realizing eqn. (15), eqn. (26) leads to two expressions for reactive conductivities of electrons and heavy particles
\[ k_{re} = \left( \frac{n D_{\text{amb}}}{N_A} \right) \left[ H_e + H_{i+1} - H_i \right] \frac{\partial x_e}{\partial T_e} \]
\[ k_{rh} = \left( \frac{n D_{\text{amb}}}{N_A} \right) \left[ H_e + H_{i+1} - H_i \right] \frac{\partial x_e}{\partial T_h} \]

The total heat flux due to conduction and diffusion in the absence of radiation can now be written in terms of gradients of temperature of electrons and heavy particles under LTE conditions as
In the most ideal case of \( \theta = \text{constant} \) (similarity of electron and heavy particle temperature fields), eqn. (29) reduces to

\[
q = - (k_h + \theta k_e) \nabla T_h = - k \nabla T_h
\]  

(30)

4.7. Discussion of thermodynamics and transport coefficients

Local thermal equilibrium, species' kinetic temperatures and state equations are derived/presented for a two-temperature plasma based on thermodynamic principles. Also derived are the transport coefficients for a multiple-ionized non-equilibrium plasma and expressions for species heat flux. Plasma composition is obtained for argon up to \( T = 10 \) at pressure one bar and electron heavy particle temperatures up to 20000° K. While Model B gives positive values of partial derivatives with respect to electron and heavy particle temperatures correctly, at least the partial derivative becomes negative at certain temperatures for model A, which results in a small heat flux due to diffusion in a direction opposite to that due to the pure conduction. A suggestion is made to handle such a situation for Model A.

Part 2. Transport properties of a two-temperature argon plasma

In view of the difficulties in obtaining experimental values over a wide range of temperatures and pressures encountered in plasma situations, we find that a theoretical method for calculating the non-equilibrium transport properties is best suited for our study. With a small perturbation in distribution function and following the method of Chapman-Enskog, solutions\(^8,9\) are reported for transport coefficients of heavies and electron subgases. We extensively use the results of Devoto\(^8\) in conjunction with the expressions developed in Part I for numerical computation of two-temperature argon plasma transport properties. The average cross-sections required for the determination of transport properties of a singly ionized two-temperature argon plasma are as in Kannappan and Bose\(^11\).

14. Heavy particle transport coefficients

We present here only the simplified expressions for heavies transport coefficients to the first approximation. These expressions are linked with the two-temperature plasma composition results in Part 1. The heavy particles (atoms and ions) are treated as a binary mixture.

Denoting the diffusivities of the pair \( i-j \) in a binary and a multi-component mixture by \( D_{ij} \) and \( D_{ij} \) respectively, the diffusion coefficient is given by the expression
\[ D_{ij} = D_{ij} \left[ 1 + \frac{x_i \left( \frac{m_i}{m_j} \right) D_{ik} - D_{ij}}{x_i D_{ik} + x_j D_{jk} + x_k D_{ij}} \right] \]

where
\[
D_{ij} = \frac{3}{16n} \left( \frac{2\pi k_B T}{m_{ij}} \right) \frac{1}{Q_{ij}}^{(1,1)}.
\]

The subscripts \( i \) and \( j \) take the following combinations in the expression for \( D_{ij} \):

\( (i, j) = (i + 1, i), (i + 1, e), (e, i) \) and \( (e, i + 1) \)

\[ D_{i+1, e} = D_{i-1, e} \left[ 1 - \frac{m_e}{m_i} \frac{D_{i-1, e}}{D_{i+1, e}} \left( 1 - \frac{D_{ele}}{D_{ele+1}} \right) \right] \]

and \( D_{emb} \) for a two-temperature plasma is given by eqn. (23) in Part 1.

Thermal conductivity and viscosity with the first approximation are calculated from Devoto\(^8\). The viscosity of the mixture has been assumed to be entirely due to the heavy particles. The ion contribution of electrical conductivity is neglected because it is of the order of \( \chi (m_e/m_h) \) times the electron contribution.

### 2.2. Electron transport coefficients

For the calculation of electron subgas transport properties, fourth approximation is used.

\[ D_{ee} = \frac{3}{2} n_e \left( \frac{2\pi k_B T_e}{m_e} \right)^{1/2} \frac{1}{Q |q|} \begin{vmatrix} q^{11} & q^{12} & q^{13} \\ q^{21} & q^{22} & q^{23} \\ q^{31} & q^{32} & q^{33} \end{vmatrix}, \text{m}^2/\text{s} \]

The electrical conductivity due to electrons is

\[ \sigma = e^2 n_e D_{ee}/k_B T_e, \text{A/Vm} \]

The expression for thermal conductivity is (reactive heat conductivity is given in Part 1).

\[ k_e = \frac{75n_e^2 k_B}{8} \left( \frac{2\pi k_B T_e}{m_e} \right)^{1/2} \frac{1}{Q |q|} \begin{vmatrix} q^{01} & q^{02} & q^{03} \\ q^{21} & q^{22} & q^{23} \\ q^{31} & q^{32} & q^{33} \end{vmatrix}, \frac{W}{\text{mk}} \]

The determinant, \( |q| \) and the expressions for elements \( q \)'s are from Devoto\(^5\). The contribution of electron to the mixture viscosity is neglected on the physical grounds that the viscosity of a species is proportional to the square root of its mass.

### 2.3. Discussion of transport properties

In figs. 5–8, only representative of the two-temperature transport properties of argon plasma is shown at \( p = 1 \) bar. Those curves for which \( \theta = 1 \) in all the figures correspond to the case of equilibrium conditions and agree with the results of Devoto\(^4\).
Figure 5 shows a uniform variation of $D_{\text{amb}}$ with $T_e$, and figs. 6 and 7 the total heavy and heavies thermal conductivity coefficients respectively. The total thermal conductivity variation of heavies is different from that of electrons as observed in figs. 6 and 7 respectively. The electron heat conductivity coefficient increases steeply up to $T_e \approx 14000^\circ K$ whereas the temperature above $14000^\circ K$, there is no appreciable increase. This is due to the fact that electrons up to $T_e \approx 14000^\circ K$, acquire high mobility and also lesser fraction of neutrals present in the mixture (refer fig. 3 $x_e$ vs $T_k$ with parameter, $T_e$). The contribution of heavies to total mixture conductivity is appreciable at lower temperatures and is negligible at higher temperatures. The mixture total thermal conductivity at lower temperatures is due entirely to the atoms owing to the large neutral particle cross-sections. Figure 8 shows the variation of electrical conductivity with electron temperature and $\theta$ as parameter. The increase in electrical conductivity beyond a certain electron temperature shown in fig. 8 with increasing $\theta$ values is due to the domination of charged particle interaction at high electron temperatures. The decrease in $\sigma$ with increasing $\theta$ values for a given $T_e$ follows from the Ramsauer effect (i.e., domination of $e-A$ interaction at low values of $\theta$). The results shown in fig. 8 also demonstrate quantitatively the possibility of achieving high value of electrical conductivity at reasonably lower gas temperatures.

The values of transport properties of argon shown in figs. 5-8 are limited to a maximum temperature of $20000^\circ K$ because $\text{Ar}^{+4}$ ion should be considered in computing the composition and transport coefficients beyond this limit.
3. Summary and conclusion

In part 1 we have discussed the LTE, the non-equipartition of energy and the multiple-temperature concept in a two-temperature plasma. Thermodynamic state equations and Saha equation are defined/discussed. Also shown are the derivations of expressions for transport coefficients of a two-temperature plasma. Variation of electron mol-
Fig. 7. Heavy particle total heat conductivity coefficient of a two-temperature argon plasma.

Fraction as a function of electron and heavy particle temperatures is studied for both models with special reference to argon at \( p = 1 \) bar and temperatures up to \( 20000^\circ\) K. The partial derivatives of \( x_e \) with respect to \( T_e \) and \( T_h \) are discussed with reference to the direction and heat flux diffusion. In Part 2 are shown the expressions of transport coefficients of electron and heavies subgases constituting a plasma mixture. The
FIG. 8. Electrical conductivity of a two-temperature argon plasma.

computed results of transport properties of two-temperature argon plasma at $p = 1$ bar are shown and discussed. Comparison with earlier results for the special case of a plasma under thermal equilibrium shows a good agreement.

Nomenclature

$a, b$ \text{variables defined in Eqn. (A. 5)}


mobility coefficient

diffusion coefficient

binary diffusion coefficient

electric field

elementary charge of electron

functions defined in Eqns. (A. 3, 4 and A. 8, 9)
molar enthalpy

specific enthalpy

Planck constant

ionization potential

ionization index, for neutrals $i = 1$
current density

Boltzmann constant

pure heat conduction coefficient

reactive heat conduction coefficient

mass of a particle

molar mass

Avagadro number

number density

pressure

heat flux vector

universal gas constant

Saha function

temperature

diffusive velocity

mole fraction

partition function

degree of ionization

energy loss factor

density

temperature ratio, $T_e/T_h$

Subscripts

c  conduction
e  electron

$\infty$  equilibrium

k  heavy particle, ion or atom

H  heavies (ions plus neutrals)
i  ionization index, for neutrals $i = 1$
s  species

r  reactive or diffusive part
References

1. KRUGER, C. H.

2. INCROPERA, F. P. AND VIEGAS, J. R.

3. BAHADORI, M. N. AND SOO, S. L.

4. EDDY, T. L., PFENDER, E. AND ECKERT, E. R. G.

5. LIU, W. S., WHITTEN, B. T. AND GLASS, I. I.

6. DRAWIN, H. W.

7. BOSE, T. K.

8. DEVOTO, R. S.

9. DEVOTO, R. S.

10. DEVOTO, R. S.

11. KANNAPPAN, D. AND BOSE, T. K.

12. KANNAPPAN, D. AND BOSE, T. K.

13. PFENDER, E. AND ECKERT, E. R. G.

14. CLARK, K. J. AND INCROPERA, F. P.

15. KERREBROCK, J. L.

16. MORSE, T. L.

17. SPITZER, L.

18. McDaniel, E. W.

19. MONTI, R. AND NAPOLITANO, L. G.

20. VEIS, S.

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Appendix A

Two-temperature Saha equation and its partial derivative expressions

The following analytical expressions of partial derivatives for model A are used in the calculation of species coefficients, and fluxes as well as in the systems of equations formulated to study the energy transfer processes.

Model A

\[ \frac{\partial x_e}{\partial T_e} = \frac{g^0}{fT_h^3} \]  
\[ \frac{\partial x_a}{\partial T_h} = -\frac{g}{fT_h} + \frac{1}{f} \left[ \frac{2.5}{T_e} \frac{\theta}{T_e} + \frac{I_e}{(k_BT_e)^2} \right] \]  

where

\[ f = \frac{\theta (x_e + a) + x_e}{x_e (x_e + a)} - \frac{\theta^2 (1 - bx_e) - b (1 - x_e) - \theta}{(1 - bx_e) [x_e (\theta - 1) + 1]} \]  
\[ g = \frac{(1 - x_e)}{[x_e (\theta - 1, 1) + 1]} - \log_e (S/x_e) \]  
\[ a = (1 - i)/i, \: b = (i + 1)/i \]

The following are the expressions for model B.

\[ \frac{\partial x_e}{\partial T_e} = \frac{g}{fT_h^3} \]  
\[ \frac{\partial x_a}{\partial T_h} = \frac{g}{fT_h} + \frac{1}{f} \left[ \frac{2.5}{T_e} \frac{1}{T_h} + \frac{I_e}{(k_BT_h)^2} \right] \]  

where

\[ f = \frac{2x_e + a}{x_e (x_e + a)} - \frac{[(\theta - 1) (1 - 2bx_e) - b}{(1 - bx_e) [x_e (\theta - 1) + 1]} \]  
\[ g = \frac{1 - x_e}{[x_e (\theta - 1) + 1]} \]  

and \( b \) from equation (A.5).